MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME: Mr. Lim

CENTRE NUMBER: S

INDEX NUMBER:

MATHEMATICS

9758/02

Paper 2

October/November 2023

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 **(a)** Find the set of values of x for which $3|x-2|<|5-2x|$. [2]

From G.C. $\{x : x \in \mathbb{R},\ 1 < x < 2.2\}$

(b) Express $\dfrac{x+25}{x^2-4x-5}+3$ as a single simplified fraction. Hence, without using a calculator, solve exactly the inequality $\dfrac{x+25}{x^2-4x-5}>-3$. [4]

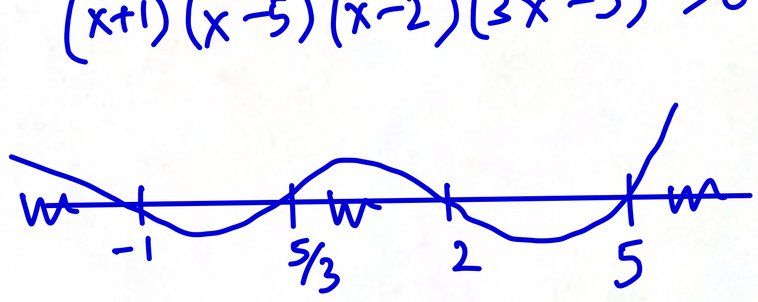
$$\frac{3x^2-12x+x-15+25}{x^2-4x-5} = \frac{3x^2-11x+10}{x^2-4x-5} = \frac{(x-2)(3x-5)}{(x+1)(x-5)}$$

$$\frac{x+25}{x^2-4x-5} > -3$$

$$\frac{x+25}{x^2-4x-5}+3 > 0$$

$$\Rightarrow \frac{(x-2)(3x-5)}{(x+1)(x-5)} > 0$$

$$(x+1)(x-5)(x-2)(3x-5) > 0$$



$x<-1$ or $\frac{5}{3}<x<2$ or $x>5$

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2 (a) Given that $y = \ln(\sec x)$, show that $\dfrac{d^3y}{dx^3} = 2\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)$. [3]

$\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$

$\frac{d^2y}{dx^2} = \sec^2 x$

$\frac{d^3y}{dx^3} = 2\sec x(\sec x \tan x)$

$= 2\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)$ #

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2 **[Continued]**

(b) Hence, or otherwise, obtain the Maclaurin expansion of y in terms of x up to and including the term in x^4. [3]

$$\frac{d^4y}{dx^4} = 2\left(\frac{d^3y}{dx^3}\right)\left(\frac{dy}{dx}\right) + 2\left(\frac{d^2y}{dx^2}\right)^2$$

when $x=0$ $\quad y=0$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 1$$

$$\frac{d^3y}{dx^3} = 0$$

$$\frac{d^4y}{dx^4} = 2$$

$$y = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \ldots$$

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(c) By putting $x = \frac{1}{4}\pi$, find an approximation for $\ln 2$ in terms of π. [2]

$$\ln\left(\frac{1}{\cos x}\right) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \ldots$$

$$\ln\left(\sqrt{2}\right) = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 + \frac{1}{12}\left(\frac{\pi}{4}\right)^4 + \ldots$$

$$\frac{1}{2}\ln 2 \approx \frac{\pi^2}{32} + \frac{\pi^4}{3072}$$

$$\ln 2 \approx \frac{\pi^2}{16} + \frac{\pi^4}{1536}$$

(d) Using your answer to part **(b)**, find an approximation to $\int_0^{\frac{1}{10}\pi} \ln(\sec x)\,dx$. Give your answer correct to 4 significant figures. [1]

$$\int_0^{\frac{1}{10}\pi} \ln(\sec x)\,dx \approx \int_0^{\frac{1}{10}\pi} \frac{1}{2}x^2 + \frac{1}{12}x^4 \,dx$$

$$= 0.0052187$$

$$\approx 0.005219$$

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3 The points A and B have position vectors $\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix}$ respectively.

The point C is such that $\overrightarrow{BC} = 2\overrightarrow{AB}$.

(a) Find the position vector of C. [2]

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$

$\begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix} + \overrightarrow{OC} = \begin{pmatrix} 4 \\ -8 \\ 6 \end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix} 5 \\ -10 \\ 14 \end{pmatrix}$

The point D has position vector $\begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix}$ and is such that $|\overrightarrow{AD}| = |\overrightarrow{BD}|$.

(b) Find the value of d. [2]

$\overrightarrow{AD} = \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ d-5 \end{pmatrix}$ $\quad \overrightarrow{BD} = \begin{pmatrix} -1 \\ 2 \\ -8 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ d-8 \end{pmatrix}$

$|\overrightarrow{AD}| = |\overrightarrow{BD}|$

$4 + (d-5)^2 = 16 + (d-8)^2$

$-10d + 25 = 12 + 64 - 16d$

$6d = 51$

$d = \frac{51}{6} = \frac{17}{2}$

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(c) Use a scalar product to find angle ADB. [3]

$\overrightarrow{AD} = \begin{pmatrix} 2 \\ 0 \\ 7/2 \end{pmatrix}$ $\overrightarrow{BD} = \begin{pmatrix} 0 \\ 4 \\ 1/2 \end{pmatrix}$

$$\cos \angle ADB = \frac{\begin{pmatrix} 2 \\ 0 \\ 7/2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ 1/2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 0 \\ 7/2 \end{pmatrix} \right| \left| \begin{pmatrix} 0 \\ 4 \\ 1/2 \end{pmatrix} \right|}$$

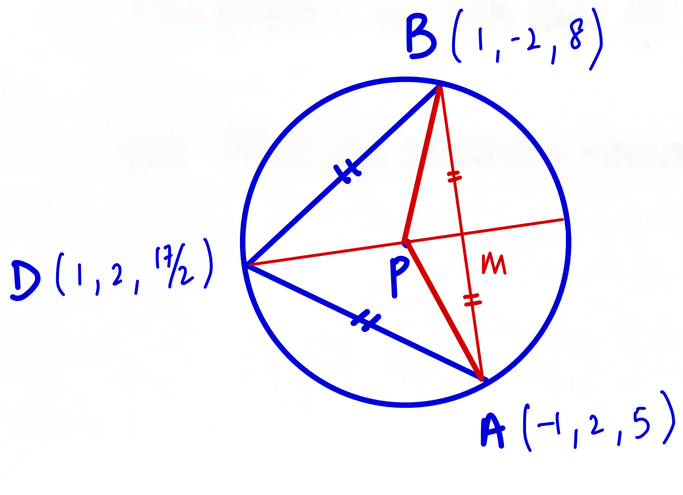
$$= \frac{\frac{7}{4}}{16\frac{1}{4}}$$

$\angle ADB = 83.8°$ #

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3 [Continued]

(d) Find exactly the position vector of the point P, where P is the centre of the circle that passes through A, B and D. [5]



Let m be the midpoint of AB

$$2\overrightarrow{OM} = \begin{pmatrix}-1\\2\\5\end{pmatrix} + \begin{pmatrix}1\\-2\\8\end{pmatrix} = \begin{pmatrix}0\\0\\13\end{pmatrix}$$

$$\overrightarrow{OM} = \begin{pmatrix}0\\0\\13/2\end{pmatrix}$$

$$\overrightarrow{MD} = \begin{pmatrix}0\\0\\-13/2\end{pmatrix} + \begin{pmatrix}1\\2\\17/2\end{pmatrix} = \begin{pmatrix}1\\2\\2\end{pmatrix}$$

$l_{PD} : r = \begin{pmatrix}1\\2\\17/2\end{pmatrix} + \begin{pmatrix}1\\2\\2\end{pmatrix}\lambda$, where $\lambda \in \mathbb{R}$

Let point P be $\begin{pmatrix}1+\lambda\\2+2\lambda\\17/2+2\lambda\end{pmatrix}$

$$\overrightarrow{BP} = \begin{pmatrix}-1\\2\\-8\end{pmatrix} + \begin{pmatrix}1+\lambda\\2+2\lambda\\17/2+2\lambda\end{pmatrix} = \begin{pmatrix}\lambda\\4+2\lambda\\1/2+2\lambda\end{pmatrix}$$

$$\overrightarrow{PD} = \begin{pmatrix}\lambda\\2\lambda\\2\lambda\end{pmatrix}$$

radius: $|\overrightarrow{BP}| = |\overrightarrow{PD}|$

$$\sqrt{\lambda^2 + (4+2\lambda)^2 + (\tfrac{1}{2}+2\lambda)^2} = \sqrt{9\lambda^2}$$

$$18\lambda = -65/4$$

$$\lambda = -65/72$$

$$\therefore \overrightarrow{OP} = \frac{1}{72}\begin{pmatrix}7\\14\\482\end{pmatrix}$$

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4 The curve C is defined by the parametric equations $x = 2t^2 + 3$ and $y = 5t - 1$, where $t \geqslant \frac{1}{5}$.

(a) Find the exact area between the curve C, the x-axis and the line $x = 21$. [3]

$\frac{dx}{dt} = 4t$

$21 = 2t^2 + 3$

$t = 3 \quad \because t \geqslant \frac{1}{5}$

Area : $\int y \, dx$

$= \int_{1/5}^{3} (5t-1)(4t) \, dt$

$= 4\left[\frac{5t^3}{3} - \frac{t^2}{2}\right]_{1/5}^{3}$

$= 4\left(\left(45 - \frac{9}{2}\right) - \left(\frac{1}{75} - \frac{1}{50}\right)\right)$

$= \frac{12152}{75}$

$= 162\frac{2}{75}$ units^2 //

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4 **[Continued]**

The curve D is defined by the parametric equations $x = 5u$ and $y = \frac{4}{u}$, where $u \neq 0$.

(b) Find a point of intersection, A, of the curves C and D. Show that there are no other points of intersection. [5]

$x = 5\left(\frac{4}{y}\right)$

$xy = 20$

Sub in

$(2t^2+3)(5t-1) = 20$

$10t^3 - 2t^2 + 15t - 23 = 0$

$(t-1)(10t^2 + At + 23) = 0$

$\therefore \quad -2 = A - 10$

$A = 8$

$10t^2 + 8t + 23 = 0$

$D: \quad b^2 - 4ac$

$= 64 - 4(10)(23)$

$= -856 < 0$

$\therefore$ $t = 1$ is the only real root, hence there is no other point of intersection.

At point A, $t = 1 \quad x = 5, \; y = 4$

$\therefore A(5, 4)$ //

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(c) Find the coordinates of the point where the tangent to curve C at the point A meets curve D for a second time. [5]

$\frac{dx}{dt} = 4t$ $\quad \frac{dy}{dt} = 5$ $\quad \therefore \left.\frac{dy}{dx}\right|_{t=1} = \frac{5}{4}$

Eq^n of tangent: $y - 4 = \frac{5}{4}(x - 5)$

$y = \frac{5}{4}x - \frac{9}{4}$

sub in $\frac{4}{u} = \frac{5}{4}(5u) - \frac{9}{4}$

$4 = \frac{25}{4}u^2 - \frac{9}{4}u$

$25u^2 - 9u - 16 = 0$

$u = 1$ or $u = -\frac{16}{25}$

coordinate $\left(-\frac{16}{5}, -\frac{25}{4}\right)$ #

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Section B: Probability and Statistics [60 marks]

5 A child's toy has 36 slots, numbered from 1 to 36. A child puts a ball into the toy, the ball falls into one of the 36 slots and the child's score is the number of that slot. The ball is equally likely to fall into any one of the slots.

Sadiq is investigating four possible events, A, B, C and D, which are defined as follows.

A the score is odd 1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 . . .
B the score is even 2 4 6 8 10 12 14 16 18 20 22 . . .
C the score is a multiple of 3 3 6 9 12 15 18 21 24 . . .
D the score is a multiple of 6 6 12 18 24 30 36

(a) (i) State which pairs of the events, if any, are mutually exclusive. [1]

A & B and A & D

(ii) Show that A and C are independent events, and state another pair of independent events. [2]

$P(A) = \frac{1}{2}$ $P(C) = \frac{1}{3}$

$A \cap C$ = odd and multiple of 3
= 3, 9, 15, 21, 27, 33

$\therefore P(A \cap C) = \frac{6}{36}$
$= \frac{1}{6}$
$= P(A) \times P(B)$

$\therefore$ A & C are independent events

B & C are also independent events.

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Sadiq notices that a ball has become stuck in the slot labelled 36, and so balls put into the toy are now falling, with equal probability, into one of only 35 slots, and the score can only be from 1 to 35.

(b) (i) State which pairs of the four events, if any, are **now** mutually exclusive. [1]

A & B and A & D

(ii) Determine whether A and C are **now** independent events. [1]

$P(A) = \frac{18}{35}$ $\quad P(C) = \frac{11}{35}$

$P(A \cap C) = \frac{11}{35} \neq \frac{18}{35} \times \frac{11}{36}$

$\therefore$ A & C are not independent events.

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6 A bag contains r red counters and b blue counters, where $r > 12$ and $b > 12$. Mei randomly removes 12 counters from the bag. The probability that there are 4 red counters among Mei's 12 counters is the same as the probability that there are 3 red counters.

(a) Show that $9r + 5 = 4b$. [2]

$$\frac{{}^{r}C_{4} \cdot {}^{b}C_{8}}{{}^{r+b}C_{12}} = \frac{{}^{r}C_{3}\ {}^{b}C_{9}}{{}^{r+b}C_{12}}$$

$$\frac{r!}{(r-4)!\,4!} \cdot \frac{b!}{(b-8)!\,8!} = \frac{r!}{(r-3)!\,3!} \cdot \frac{b!}{(b-9)!\,9!}$$

$$(r-3)9 = 4(b-8)$$

$$9r - 27 = 4b - 32$$

$$9r + 5 = 4b \quad //$$

The probability that there are 3 red counters among Mei's 12 counters is $\frac{5}{3}$ times the probability that there are 2 red counters.

(b) Derive an equation similar to the equation in part **(a)** and hence find the probability that just one of the 12 counters removed is red. [6]

$$\frac{{}^{r}C_{3}\ {}^{b}C_{9}}{{}^{r+b}C_{12}} = \left(\frac{5}{3}\right)\frac{{}^{r}C_{2}\ {}^{b}C_{10}}{{}^{r+b}C_{12}}$$

$$\frac{r!}{(r-3)!\,3!} \cdot \frac{b!}{(b-9)!\,9!} = \frac{5}{3}\,\frac{r!}{(r-2)!\,2!} \cdot \frac{b!}{(b-10)!\,10!}$$

$$(r-2)\,10 = 3(b-9)\left[\frac{5}{3}\right]$$

$$10r - 20 = 5b - 45$$

$$\left.\begin{aligned} 10r - 5b &= -25 \\ 9r - 4b &= -5 \end{aligned}\right\} \text{ solve}$$

$$r = 15 \quad b = 35$$

Prob: $$\frac{{}^{15}C_{1}\ {}^{35}C_{11}}{{}^{50}C_{12}} = 0.05155 \approx 0.0516$$

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6 [Continued]

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7 The numbers of mobile phone subscriptions in Singapore, y million, for certain years from 2004 are given in the following table. The variable x is the number of years after a base year of 2000.

Year	2004	2006	2008	2010	2012	2014	2016	2018
Number of years after base year, x	4	6	8	10	12	14	16	18
Number of subscriptions, y million	3.99	4.79	6.41	7.38	8.07	8.10	8.46	8.39

Ling thinks that the number of mobile phone subscriptions in Singapore can be modelled by one of the formulae

$$y = ax + b, \qquad e^y = cx + d,$$

where a, b, c and d are constants.

(a) Find, correct to 4 decimal places, the value of the product moment correlation coefficient

(i) between x and y, [1]

$r = 0.9281$

(ii) between x and e^y. [1]

$r = 0.9697$

(b) Explain which of Ling's models, $y = ax + b$ or $e^y = cx + d$, gives a better fit to the data and find the equation of the regression line for this model. [3]

$e^y = cx + d$ is a better fit because $|r|$ is closer to 1.

From G.C. $e^y = 376x - 1880$

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(c) Use the equation of the regression line to estimate the number of mobile phone subscriptions in 2024. Explain whether your estimate is reliable. [2]

$e^y = 375.615x - 1881.4818$

$x = 24$

$e^y = 7133.2836$

$y = 8.8725$

≈ 8.87 million subscriptions

The estimate is not reliable because 2024 is out of the data range, and from the data, we can see the number of subscriptions dropped from 2016 to 2018. If we use this line, we assuming the number of subcription will keep increasing with time, which is incorrect.

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8 (a) A company has a new machine designed to fill bags with, on average, 1 kg of granulated sugar. The production manager wishes to investigate if the machine is adjusted correctly. He intends to take a sample of bags and carry out a hypothesis test.

(i) State null and alternative hypotheses for the manager's test, defining any parameters you use. [2]

null hypothesis $H_0 : \mu = 1$

alternative hypothesis $H_1 : \mu \neq 1$

μ is the population mean weight of granulated sugar.

The production manager decides to take the first bag of sugar produced each morning and the first bag of sugar produced each afternoon, in a 5-day working week, to form a sample of 10 bags for the test.

(ii) Give **two** reasons why the production manager's sample is not suitable for a z-test. [2]

(1) Since the distribution of the weight of each bag of sugar is unknown, hence taking a sample of 10 bags of the sugar is insufficient for the sample mean weight to follow a normal distribution.

(2) Selecting the 1st bag in the morning and afternoon also means the bags are not selected at random. Hence, the weight may be more consistent resulting in a biased testing results.

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(b) The company has a different machine which fills larger bags with, on average, 2 kg of granulated sugar. One of the company's sales representatives has reported that some customers suspect the machine is no longer set correctly, and that the average mass of sugar in the bags may in fact be less than 2 kg. The production manager decides to carry out a hypothesis test at the 2.5% level of significance with a suitable sample of 40 bags of sugar. Summary data for the mass, x kg, of sugar in these bags is as follows.

$$n = 40 \qquad \Sigma x = 78.88 \qquad \Sigma x^2 = 155.6746$$

(i) State the hypotheses and find the critical region for this test. [5]

$\bar{x} = \frac{\Sigma x}{n} = 1.972$ $\quad s^2 = \frac{1}{39}\left(155.6746 - \frac{78.88^2}{40}\right)$

$= \frac{79}{25000}$

$H_0 : \mu = 2$

$H_1 : \mu < 2$

Under H_0, since n is large, by C.L.T. $\bar{X} \sim N\left(2, \frac{79/25000}{40}\right)$ approx

Test Stat : $z = \frac{\bar{x} - 2}{\sqrt{\frac{79/25000}{40}}}$

critical region : Reject H_0 if $\frac{\bar{x} - 2}{\sqrt{\frac{79/25000}{40}}} < -1.9599$

$\bar{x} < 1.982$

$\Rightarrow \quad 0 < \bar{x} \leq 1.98$

(ii) State the conclusion of the test in the context of the question. [2]

Reject H_0 and conclude that there is sufficient evidence at 2.5% level of significance that the mean weight of sugar is less than 2 kg.

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9 In this question, you should state the parameters of any distributions you use.

A company produces wooden planks of two different lengths, Regular and Long. The lengths, in metres, of the Long planks follow the distribution $N(1.82, 0.2^2)$.

(a) Find the probability that the length of a randomly chosen Long plank is less than 1.79 m. [1]

$L \sim N(1.82, 0.2^2)$

$P(L < 1.79) = 0.44038 \approx 0.440$ #

(b) Find the probability that the total length of 8 randomly chosen Long planks is greater than 14.5 m. [3]

$L_1 + L_2 + \ldots + L_8 \sim N(14.56, 0.32)$

$P(L_1 + \ldots + L_8 > 14.5) = 0.5422$

≈ 0.542 #

The lengths, in metres, of the Regular planks follow the distribution $N(1.22, 0.3^2)$.

(c) Sylvio buys 120 of the Regular planks. Calculate the expected number of these planks that are longer than 1.25 m. [2]

$R \sim N(1.22, 0.3^2)$

$P(R > 1.25) = 0.460172$

Expected # : 55.22 $\therefore$ 55 planks.

(d) Find the probability that the total length of 10 randomly chosen Long planks differs by less than 0.65 m from the total length of 16 randomly chosen Regular planks. [3]

$W = L_1 + \ldots + L_{10} \sim N(18.2, 0.4)$

$P = R_1 + \ldots + R_{16} \sim N(19.52, 1.44)$

$W - P \sim N(-1.32, 1.84)$

$P(|W - P| < 0.65) = P(-0.65 < W - P < 0.65)$

$= 0.2374$

≈ 0.237

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The company finds that there is a demand for Short planks. These planks are produced by cutting the Long planks into three exactly equal lengths, or the Regular planks into two exactly equal lengths.

(e) Find the probability that the length of a randomly chosen Short plank made from a Long plank is greater than the length of one made from a Regular plank. You should ignore any wastage caused by cutting the plank. [4]

$$A = \frac{L}{3} \sim N\left(\frac{91}{150}, \frac{1}{225}\right)$$

$$B = \frac{R}{2} \sim N\left(\frac{61}{100}, \frac{9}{400}\right)$$

$$(A-B) \sim N\left(-\frac{1}{300}, \frac{97}{3600}\right)$$

$$P(A-B>0) = 0.49189$$

$$\approx 0.492 \quad \#$$

(f) Without doing any detailed calculation, explain how your answer to part **(e)** would change if each cut of a plank caused a small amount of wastage. [1]

The probability will be lower as longer planks require more cut than regular planks, hence more wastage and so the probability that the short planks is longer than the one made from regular planks will be lower.

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10 **(a)** A small company makes 50 glass ornaments each working day. Some of the ornaments turn out to be faulty.

(i) State, in the context of the question, two assumptions needed for the number of faulty ornaments made in a day to be well modelled by a binomial distribution. [2]

(1) The probability that the ornaments turn out to be faulty is constant for each ornaments.

(2) The event that an ornament turn out to be faulty is independent of the other ornament.

Assume now that the number of faulty ornaments produced each day has the distribution B(50, 0.04).

(ii) Show that the numerical values of the mean and variance of this distribution differ by 0.08. [1]

$X \sim B(50, 0.04)$

$E(X) = 50(0.04) = 2$

$Var(X) = 50(0.04)(1-0.04) = 1.92$

$\therefore$ difference : $2 - 1.92 = 0.08$ *

(iii) Find the probability that no more than 2 faulty ornaments are produced on a randomly chosen working day. [1]

$P(X \leq 2)$

$= 0.676714$

≈ 0.677 *

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(iv) Find the probability that no more than 2 faulty ornaments are produced on at least 3 days in a randomly chosen 5-day working week. State the distribution you use. [3]

let W be r.v. denoting # of days in a 5-day week that no more than 2 faulty ornaments are produced.

$W \sim B(5, 0.676714)$

$P(W \geq 3) = 1 - P(W \leq 2)$

$= 0.80478$

≈ 0.805 #

(v) Find the probability that no more than 10 faulty items are produced in a randomly chosen 5-day working week. State the distribution you use. [2]

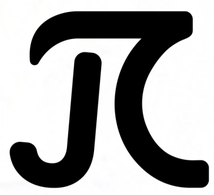
Let Y denote # of faulty ornaments in 5 days.

$Y \sim B(250, 0.04)$

$P(Y \leq 10) = 0.58305$

≈ 0.583 #

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10 [Continued]

(b) The company also makes pens which are sold in randomly packed boxes of one hundred pens. The probability of a pen being **not** faulty is p, where $0 < p < 1$.

For quality control purposes, a random sample of pens from each box is tested. Mr Lu and Mrs Ming carry out the tests but they use different methods.

Mr Lu tests a random sample of 6 pens from a box. If there are no faulty pens or only 1 faulty pen the box is accepted.

Mrs Ming tests a random sample of 3 pens from a box.

- If there are no faulty pens in her sample the box is accepted.
- If there are 2 or 3 faulty pens in her sample the box is rejected.
- If there is 1 faulty pen in her sample she takes a second random sample of 3 pens. She accepts the box if there are no faulty pens in this second sample.

Show algebraically that Mrs Ming accepts a greater proportion of boxes than Mr Lu does. [6]

$X \sim B(6, 1-p)$

Prob Mr Lu accept : $P(X \leq 1) = P(X=0) + P(X=1)$

$= p^6 + 6(1-p)(p)^5$

$Y \sim B(3, 1-p)$

Prob Mrs Ming accept : $P(Y=0) + P(Y=1)P(Y=0)$

$= p^3 + 3(1-p)(p^2)\,p^3$

$= p^3 + 3(1-p)p^5$

$p^3 + 3(1-p)p^5 - [p^6 + 6(1-p)p^5] = p^3 - p^6 - 3(1-p)p^5$

$= p^3 - p^6 - 3p^5 + 3p^6$

$= 2p^6 - 3p^5 + p^3 > 0$ for $0<p<1$ from G.C.

∴ Mrs Ming accepts more boxes

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