1 a)

$$\left.\begin{aligned} 8c + 11t + 5b &= 114 \\ 5c + 14t + 7b &= 112 \\ 9c + 9t + 4b &= 110 \end{aligned}\right\} \quad \begin{aligned} c&=7 \\ t&=3 \\ b&=5 \end{aligned}$$

b) $7x + 3y + 5z = 113 \qquad x, y, z \geq 6 \;,\; x \neq y \neq z \;\&\; z > y$

by observation

$x = 6 \;,\; y = 7 \;,\; z = 10$

2) a) $\vec{PQ} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$

$\vec{QR} = \frac{4}{3} \vec{PQ} = \begin{pmatrix} 4 \\ 4 \\ 8 \end{pmatrix} = \vec{OR} - \vec{OQ}$

$\therefore \quad \vec{OR} = \begin{pmatrix} 5 \\ 10 \\ 14 \end{pmatrix}$

b) $\vec{SQ} = \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix}$

$\begin{pmatrix} 2 \\ 8 \\ 6-c \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0 \qquad$ so $2 + 8 + 12 - 2c = 0$

$c = 11$

c) $\vec{PS} = \begin{pmatrix} -1 \\ -2 \\ 11 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix}$

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -5 \\ 11 \end{pmatrix}}{\left|\begin{matrix} 1 \\ 1 \\ 2 \end{matrix}\right| \left|\begin{matrix} 1 \\ -5 \\ 11 \end{matrix}\right|} = \frac{18}{\sqrt{6}\sqrt{147}} \qquad \therefore \theta = 52.69 \approx 52.7^\circ$$

3 a) $\frac{1}{y^2}\frac{dy}{dx} = \sin 3x$

$$-\frac{1}{y} = -\frac{1}{3}\cos 3x + c$$

$$\frac{1}{y} = \frac{\cos 3x + c}{3}$$

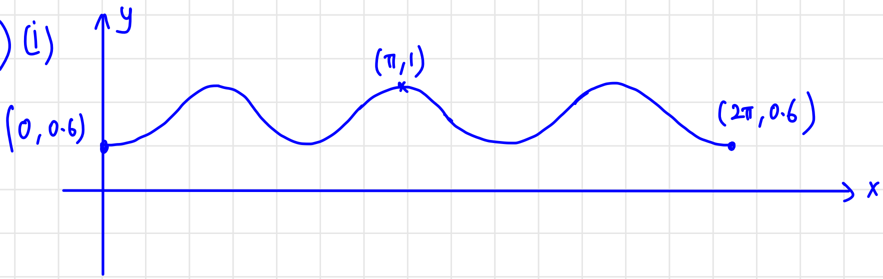
$$y = \frac{3}{c + \cos 3x}$$

sub in $x=\pi$ $y=1$

$c - 1 = 3 \quad \therefore c = 4$

$$f(x) = \frac{3}{4+\cos 3x} \qquad \therefore A=3, \ B=4$$

b) (i)



$y=1$ & $y=0.6$

(ii) $x = \frac{k\pi}{3} \quad k \in \mathbb{Z}$

(iii) $f(x+\pi) = \frac{3}{4+\cos 3(x+\pi)} = \frac{3}{4+\cos 3x \cos 3\pi - \sin 3x \sin 3\pi} = \frac{3}{4-\cos 3x}$

c) $\frac{d^2y}{dx^2} = 2y\left(\frac{dy}{dx}\right)\sin 3x + 3\cos 3x\, y^2$

$$= 2y^3 \sin^2 3x + 3\cos 3x\, y^2$$

$P(x) = 3\cos 3x \qquad Q(x) = 2\sin^2 3x$

4 a) $V = \frac{1}{3}\pi(15)^2(45-15) = 2250\pi$

time : $2250\pi \div 10\pi = 225$ seconds.

b) $V = \frac{1}{3}\pi(45h^2 - h^3)$

$\frac{dV}{dh} = \frac{1}{3}\pi(90h - 3h^2)$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$10\pi = \frac{1}{3}\pi(90(12) - 3(144))\left(\frac{dh}{dt}\right)$

$\therefore \frac{dh}{dt} = \frac{5}{108}$ cm/s

c) $\frac{dV}{dt} = 10\pi t$

$V = 5\pi t^2 + c$

$t = 0, \ V = 0 \quad$ so $c = 0$

$5\pi t^2 = 972\pi$

$t = \sqrt{\frac{972}{5}} \quad \because t \geq 0$

≈ 13.9 sec

d) $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$10\pi t = \pi(30h - h^2)\left(\frac{dh}{dt}\right)$

$\frac{dh}{dt} = \frac{10\sqrt{\frac{972}{5}}}{30(9) - 81} = 0.7377$

≈ 0.738 //

$972 = \frac{1}{3}(45h^2 - h^3)$

$0 = 45h^2 - h^3 - 2916$

$x = 43.455, \ 9, \ -7.455$

rej $\qquad$ rej

5 a) $\dfrac{{}^{3}C_{1}\,{}^{5}C_{1} + {}^{5}C_{1}\,{}^{7}C_{1} + {}^{7}C_{1}\,{}^{3}C_{1}}{{}^{15}C_{2}} = \dfrac{71}{105}$ //

b) $\dfrac{12}{15} \times \dfrac{11}{14} \times \dfrac{10}{13} \times \dfrac{9}{12} \times \dfrac{8}{11} \times \dfrac{3}{10} = \dfrac{36}{455}$

$\therefore$ n is 6.

6) a) 0.05

b) $P(X<7) = P\left(Z < \dfrac{7-\mu}{1.8}\right) = 0.05$

$\dfrac{7-\mu}{1.8} = -1.64485$

$\mu = 9.9607 \approx 9.96$

c) $P(Y>7) = \dfrac{0.38}{0.95} = 0.4$

$P\left(Z > \dfrac{7-\lambda}{2.8}\right) = 0.4$

$\dfrac{7-\lambda}{2.8} = 0.253347$

$\lambda = 6.2906 \approx 6.29$

d) $P(Y>a) = 0.8$

$a = 3.934 \approx 3.93$

7 a) $\dfrac{8!}{2!\,2!} = 10080$

b) $3\left(\dfrac{5!}{2!\,2!}\right)\left({}^{4}C_{1}\right)(2) = 720$ 2 3 3 5 7 7 8

c) $\dfrac{6!}{2!\,2!}\,3! = 1080$ $\boxed{248}$ 3 3 5 7 7

d) odd & even tgt: $2!\left(\dfrac{5!}{2!\,2!}\right)\left(3!\right) = 360$ $\boxed{33577}\ \boxed{248}$

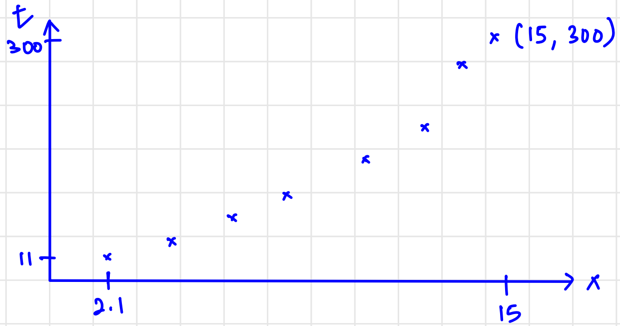
$$\text{Prob} = P(\text{odds fgt} \mid \text{even fgt}) = \frac{P(\text{odd \& even fgt})}{P(\text{even fgt})}$$

$$= \frac{360/10080}{1080/10080} = \frac{1}{3}$$

8 a) The correlation coefficient shows there is negligible correlation between the performance of high jump & long jump.

However, from the scatter diagram, we observed that the data are cluster together at the top right corner of the graph. This suggest that when an athlete is good at high jump, his long jump is relatively good as well.

b)(i)



Scatter diagram shows a curvilinear relationship.

(ii)(A)

$t = ax + b$	$t = cx^2 + d$
$a = 21.38522$	$c = 1.24969$
$b = -66.22077$	$d = 0.875809$
$r = 0.95751$	$r = 0.9904$

$\therefore$ $t = cx^2 + d$ is a better model as the correlation coefficient is closer to 1 & the data points on the scatter diagram seems to lies on a concave upwards curve.

(B) $t = 1.25x^2 + 0.876$

(iii) (A) $t = 1.25x^2 + 0.876$

when $x = 11$, $t = 152.088 \approx 152$

(B) r is closed to 1 and $x = 11$ is within the data range.

9) a) The event that a randomly chosen refrigerator is faulty is independent of other refrigerators.
The probability of a randomly chosen refrigerator is faulty is constant for each observation.

b) $X \sim B(90, 0.02)$

$P(X>1) = 1 - P(X \le 1) = 0.53957 \approx 0.540$

c) Let Y denote "more than 1 faulty refrigerator" in a 5-day week

$Y \sim B(5, 0.53957)$

$P(Y<2) = 0.14194 \approx 0.142$

d) Let W denote number of faulty refrigerators in 5-day week.

$W \sim B(450, 0.02)$

$P(W<10) = 0.58741 \approx 0.587$

e) $P(X=2)P(Y=0) + P(X=1)P(Y=1) + P(X=0)P(Y=2)$

$= 0.04353727 + 0.08896067 + 0.04418991 5$

$= 0.1766 \approx 0.177$

10) a) They are 2 different machines

b) $P \sim N(2.2, 0.1^2) \quad Q \sim N(2.1, 0.05^2)$

$P - Q \sim N(0.1, \frac{1}{80})$

$P(P>Q) = P(P-Q>0)$

$= 0.8144 \approx 0.814$

c) $P_1 + P_2 + P_3 \sim N(6.6, 0.03) \quad Q_1 + \dots + Q_5 \sim N(10.5, 0.0125)$

$P_1 + P_2 + P_3 + Q_1 + \dots + Q_3 \sim N(17.1, 0.0425)$

$P(P_1 + \dots + Q_3 > 17) = 0.68618 \approx 0.686$

d) $H_0 : \mu = 2.2$ $\quad$ H_0 is null hypothesis

$H_1 : \mu \neq 2.2$ $\quad$ H_1 is alternative hypothesis

μ is population mean mass of flour.

e) $\bar{x} = 2.15$ $\quad s^2 = \frac{1}{29}\left(1.11 - \frac{4.5^2}{30}\right)$

$= 0.015$

f) Under H_0, $n=30$, since n is large, by C.L.T $\bar{X} \sim N\left(2.2, \frac{0.015}{30}\right)$ approx

Test statistic : $Z = \dfrac{2.15 - 2.2}{\sqrt{0.015}/\sqrt{30}} = -2.236$

p-value $= 2P(Z < -2.236) = 0.0253 < 0.05$

Hence reject H_0 and conclude that there is sufficient evidence at 5% level of significance that the mean mass of the packets differs from 2.2 kg.

g) After the machine has been adjusted, the mass of the packets may or may not follow normal distribution. Hence, with 30 packets, by C.L.T, the mean mass of the packets will follow normal distribution.

h) If the sample is not chosen randomly, that is, every packet has an equal chance of being chosen, then the masses of the packets may be skewed and resulting in an erroneous conclusion.

solutions served as a suggestion only

Section A: Pure Mathematics [40 marks]

1 In a game, players have to collect tokens in the form of cars, trains and boats. A car is worth c points, a train t points and a boat b points. At the end of a game the total score of each player is worked out and the player with the highest score wins.

Pritti, Quentin, Ria and Sam played a game. At the end of the game:

- Pritti had 8 cars, 11 trains and 5 boats and scored 114 points.
- Quentin had 5 cars, 14 trains and 7 boats and scored 112 points.
- Ria had 9 cars, 9 trains and 4 boats and scored 110 points.
- Each player scored a different number of points.
- Sam came second.

(a) Find the values of c, t and b. [3]

At the end of the game, Sam had at least 6 of each type of token, different numbers of each type of token, and more boats than trains.

(b) Determine how many of each type of token Sam had at the end of the game. [You are given that there is only one solution.] [3]

2 The points P, Q and R are collinear. P and Q have position vectors $\begin{pmatrix}-2\\3\\0\end{pmatrix}$ and $\begin{pmatrix}1\\6\\6\end{pmatrix}$ respectively. The point R is such that $3\overrightarrow{QR} = 4\overrightarrow{PQ}$.

(a) Find the position vector of R. [3]

The point S has position vector $\begin{pmatrix}-1\\-2\\c\end{pmatrix}$, where c is a constant and $\overrightarrow{SQ}$ is perpendicular to $\overrightarrow{PQ}$.

(b) Find the value of c. [2]

(c) Use a scalar product to find the angle between $\overrightarrow{PS}$ and $\overrightarrow{PQ}$. [3]

3 The function $y = \mathrm{f}(x)$ is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y^2 \sin 3x$, and $y = 1$ when $x = \pi$.

(a) Show that $\mathrm{f}(x) = \dfrac{A}{B + \cos 3x}$, where A and B are constants to be determined. [4]

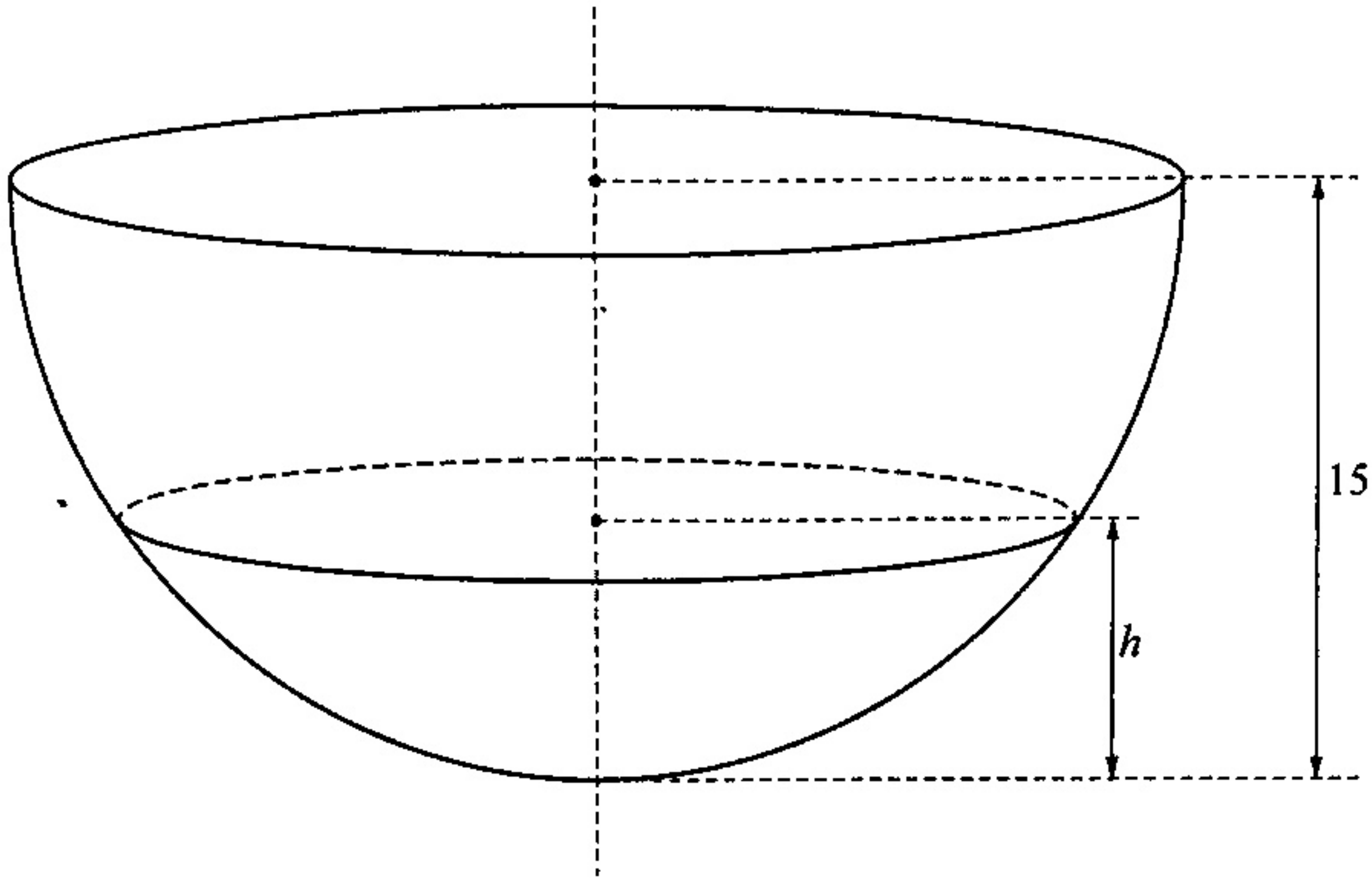
(b) (i) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant 2\pi$. State the values of c for which the line $y = c$ is a tangent to the curve. [3]

(ii) State the equations of the axes of symmetry of $y = \mathrm{f}(x)$. [2]

(iii) Find an expression for $\mathrm{f}(x + \pi)$, giving your answer in terms of $\cos 3x$. [1]

(c) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{P}y^2 + \mathrm{Q}y^3$, where P and Q are functions of x to be determined. [3]

4



The diagram shows a hemispherical bowl, of inside radius 15 cm, fixed so its circular rim is horizontal. When the depth of water in the bowl is h cm, the volume, V cm^3, of water in the bowl is given by $V = \frac{1}{3}\pi h^2(45 - h)$.

The bowl is initially empty, and water is poured into the bowl at the constant rate of 10π cm^3 per second.

(a) Find the time taken to fill the bowl. [2]

(b) Find the exact rate at which the depth of water is increasing when the depth of water is 12 cm. [4]

The bowl is emptied, and water is poured into the bowl at the variable rate of $10\pi t$ cm^3 per second, where t seconds is the time from when pouring started.

(c) Find the time taken to pour 972π cm^3 of water into the bowl. [3]

(d) Find the rate at which the depth of water is increasing when the volume of water in the bowl is 972π cm^3. [4]

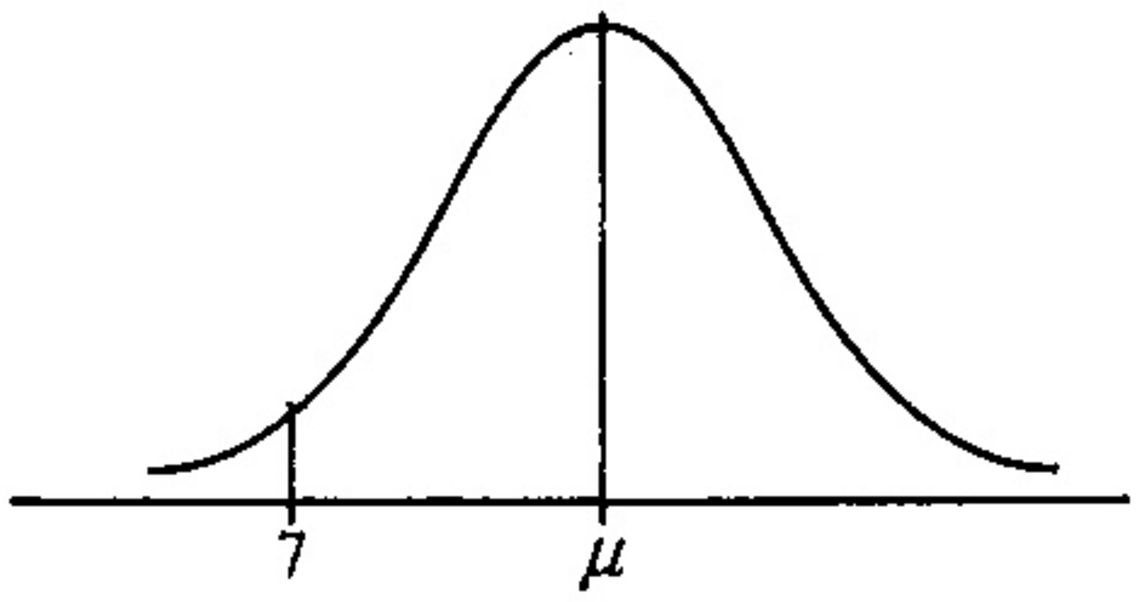
Section B: Probability and Statistics [60 marks]

5 Anya and Ben each have a bag containing 3 red counters, 5 blue counters and 7 yellow counters.

(a) Anya takes 2 counters at random from her bag without replacement. Find the probability that the 2 counters are of different colours. [3]

(b) Ben takes counters at random from his bag one at a time without replacement. The probability that his first red counter is the nth counter he takes is $\frac{36}{455}$. Find the value of n. [2]

6



The random variable X is normally distributed with mean μ, as shown in the diagram. The shaded region between 7 and μ represents 45% of the distribution.

(a) Find $P(X < 7)$. [1]

The standard deviation of X is 1.8.

(b) Find the value of μ. [2]

The random variable Y has the distribution $N(\lambda, 2.8^2)$. The random variables X and Y are independent, and $P((X > 7) \cap (Y > 7)) = 0.38$.

(c) Find the value of λ. [3]

(d) Given that $P(Y > a) = 2P(Y > 7)$, find the value of a. [2]

7 Fred has 8 cards with the following digits on them.

2 3 3 4 5 7 7 8

Fred places the 8 cards in a row.

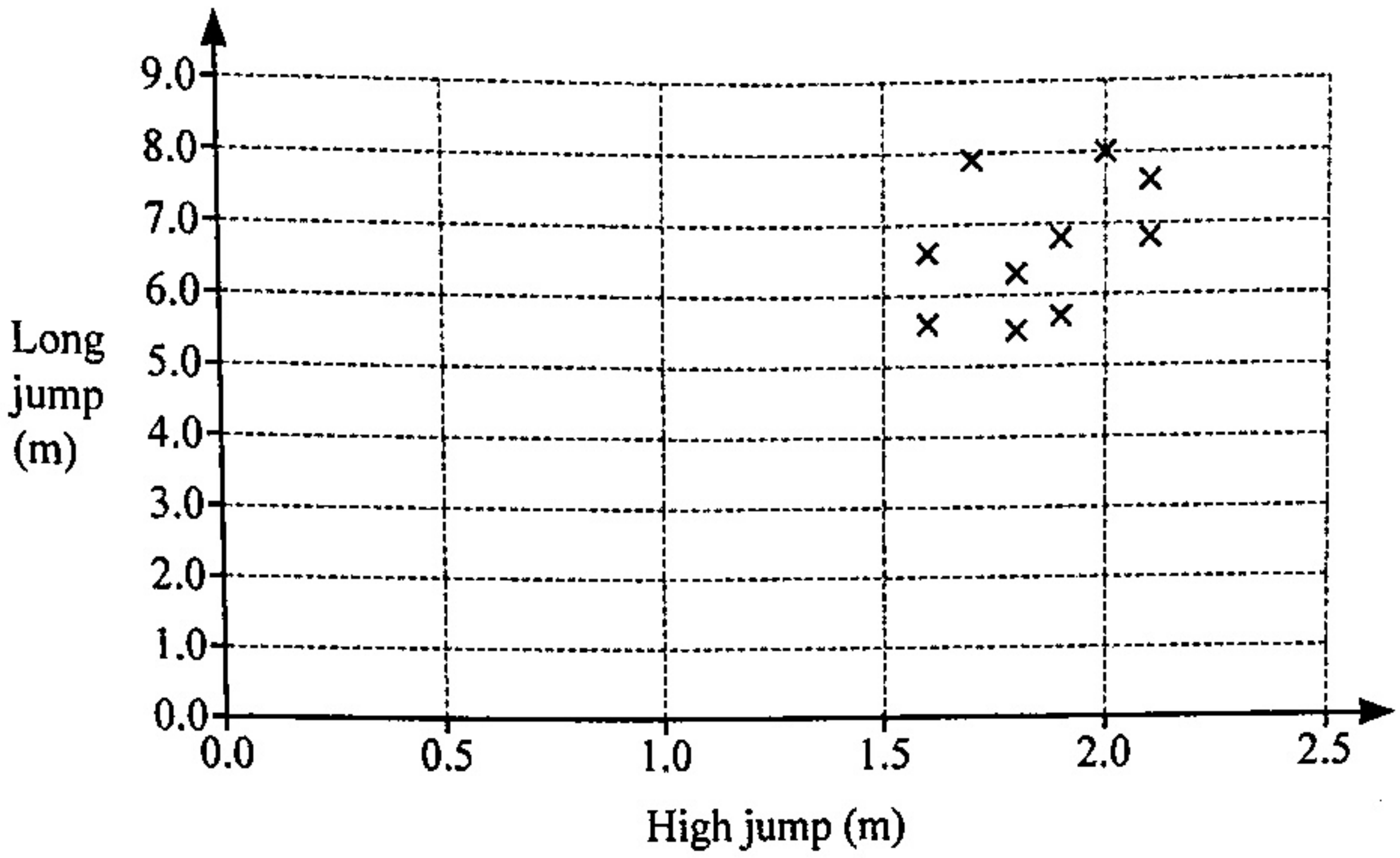
(a) Find the number of different arrangements that can be made. [2]

(b) Find the number of different arrangements that can be made which start and end with an even digit and have no two even digits next to each other. [3]

(c) Find the number of different arrangements that can be made in which the even digits are all together. [2]

(d) Given that the even digits are all together, find the probability that the odd digits are also all together. [1]

8 **(a)** Dilip is investigating how athletes perform in related events. He collects data about the best performances of 10 randomly chosen athletes in high jump and long jump. The results, in metres, are shown in the scatter diagram. The correlation coefficient is 0.39.



What do the scatter diagram and correlation coefficient suggest about the relationship between best performance in high jump and long jump amongst athletes taking part in both events? Justify your answer. [2]

(b) Each evening, after work, Dilip goes for a run. The table below shows the distance he runs, x km, and the time he takes, t minutes, for a random sample of 8 runs.

x	2.1	3.9	6.2	7.8	10	12.5	13.8	15
t	11	26	53	75	119	165	246	300

(i) Sketch a scatter diagram of the data. State what the scatter diagram suggests about the relationship between x and t. [2]

(ii) **(A)** Determine which of the following models is the better fit to the data where a, b, c and d are constants.

$$t = ax + b \qquad t = cx^2 + d$$ [2]

(B) Find the regression equation for the model identified in part **(ii)(A)**. [2]

(iii) **(A)** Use the regression equation found in part **(ii)(B)** to estimate the time Dilip would take for a run of 11 km. [1]

(B) Give two reasons why your estimate is reliable. [2]

9 A company produces a variety of domestic appliances. One of Alan's jobs is to test a fixed number of randomly chosen refrigerators each day to see if any are faulty. The number found to be faulty each day is denoted by X.

(a) State, in context, two assumptions needed for X to be well-modelled by a binomial distribution. [2]

Assume now that X has the distribution $\mathrm{B}(90, 0.02)$.

(b) Find the probability that, on a randomly chosen day, Alan finds more than one faulty refrigerator. [2]

(c) Find the probability that, in a randomly chosen 5-day working week, Alan finds more than one faulty refrigerator on fewer than two days. [2]

(d) Find the probability that, in a randomly chosen 5-day working week, Alan finds fewer than 10 faulty refrigerators in total. [2]

The company also makes washing machines. Alan also tests a fixed number of randomly chosen washing machines each day. The number of washing machines found to be faulty each day is denoted by Y. The distribution of Y is $\mathrm{B}(60, 0.03)$. The random variables X and Y are independent.

(e) Find the probability that, on a randomly chosen day, the total number of refrigerators and washing machines Alan finds to be faulty is exactly two. [3]

10 In a food processing plant, two different machines are used to produce packets of flour. Over a long period of time it has been established that the masses, in kg, of packets produced by Machine P follow the distribution $\mathrm{N}(2.2, 0.1^2)$ and the masses, in kg, of packets produced by Machine Q follow the distribution $\mathrm{N}(2.1, 0.05^2)$. It is assumed that these two distributions are independent.

(a) Explain why, in the context of the question, it is reasonable to assume that the two distributions are independent. [1]

(b) Find the probability that a randomly chosen packet produced by Machine P has a greater mass than a randomly chosen packet produced by Machine Q. [3]

(c) Find the probability that 3 randomly chosen packets produced by Machine P and 5 randomly chosen packets produced by Machine Q have a total mass greater than 17 kg. [3]

Following an adjustment to Machine P, the production manager wishes to test if the mean mass of packets produced by that machine now differs from 2.2 kg.

(d) State hypotheses for the production manager's test, defining any parameter that you use. [2]

The production manager finds the masses of a random sample of 30 packets produced by Machine P. The masses, x kg, are summarised below.

$$\Sigma(x-2) = 4.5 \qquad \Sigma(x-2)^2 = 1.11$$

(e) Calculate unbiased estimates of the population mean and variance of the masses of packets after Machine P has been adjusted. [2]

(f) Carry out the production manager's test at the 5% level of significance. Show the values you use to carry out the test and give your conclusion in the context of the question. [4]

(g) Explain why the production manager would have less confidence in the conclusion of the hypothesis test if a sample of fewer than 30 packets had been used. [1]

(h) Explain why the production manager would have less confidence in the conclusion of the hypothesis test if the sample had not been chosen randomly. [1]

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