Suggested Answers

MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME: Mr. Lim Chu Wei

CENTRE NUMBER: S

INDEX NUMBER:

MATHEMATICS 9758/01

Paper 1

October/November 2021

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Suggested Answers

BLANK PAGE

Suggested Answers

1 A function f is defined by $f(x) = ax^3 + bx^2 + cx + d$. The graph of $y = f(x)$ passes through the points $(1, 5)$ and $(-1, -3)$. The graph has a turning point at $x = 1$, and $\int_0^1 f(x)\,dx = 6$.

Find the values of a, b, c and d. [5]

$f(1) = a+b+c+d = 5$ — ①

$f(-1) = -a+b-c+d = -3$ — ②

$f'(x) = 3ax^2 + 2bx + c$

$f'(1) = 3a + 2b + c = 0$ — ③

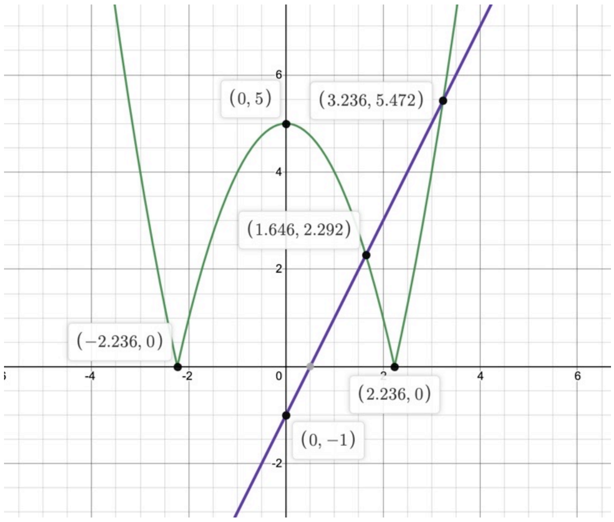
$\int_0^1 f(x)\,dx = \left[\frac{1}{4}ax^4 + \frac{1}{3}bx^3 + \frac{1}{2}cx^2 + dx\right]_0^1 = 6$

$\Rightarrow \frac{1}{4}a + \frac{1}{3}b + \frac{1}{2}c + d = 6$ — ④

From GC: $a = 4$ $b = -6$ $c = 0$ $d = 7$

Suggested Answers

2 **(a)** Sketch, on the same axes, the graphs of $y = |x^2 - 5|$ and $y = 2x - 1$. [2]



(b) Find the exact solutions of $|x^2 - 5| = 2x - 1$. [4]

See graph

Suggested Answers

3 A curve has equation $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 3$, for $x > 0$ and $y > 0$.

(a) Show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{2}}$. [2]

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\left(\frac{dy}{dx}\right) = 0$$

$$y^{-1/2}\left(\frac{dy}{dx}\right) = -x^{-1/2}$$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/2} \quad \text{Shown}$$

(b) Find the equation of the normal to the curve at the point where $x = 1$. [4]

$$\left.\frac{dy}{dx}\right|_{x=1} = -\left(\frac{4}{1}\right)^{1/2} = -2$$

$$y^{1/2} = 2$$
$$y = 4$$

Grad of normal: $\frac{1}{2}$

$$y - 4 = \frac{1}{2}(x-1)$$

$$y = \frac{1}{2}x - \frac{7}{2}$$ *

Suggested Answers

4 **Do not use a calculator in answering this question.**

The complex number z is given by

$$z = \frac{\left(\cos\left(\frac{1}{16}\pi\right) + \mathrm{i}\sin\left(\frac{1}{16}\pi\right)\right)^2}{\cos\left(\frac{1}{8}\pi\right) - \mathrm{i}\sin\left(\frac{1}{8}\pi\right)}.$$

(a) Find $|z|$ and $\arg(z)$. Hence find the value of z^2. [3]

by observation

$|z| = 1$

$$z = \frac{\left(e^{\frac{\pi}{16}i}\right)^2}{e^{-\frac{\pi}{8}i}} = e^{\left(\frac{\pi}{8}+\frac{\pi}{8}\right)i} = e^{\frac{\pi}{4}i}$$

$\arg z = \frac{\pi}{4}$

$\therefore z^2 = e^{\frac{\pi}{2}i}$ #

(b) **(i)** Show that

$$(\cos\theta + \mathrm{i}\sin\theta)(1 + \cos\theta - \mathrm{i}\sin\theta) = 1 + \cos\theta + \mathrm{i}\sin\theta.$$ [2]

$e^{i\theta}(1+e^{-i\theta})$

$= e^{i\theta}+1$

$= 1+\cos\theta + i\sin\theta$ #

(ii) Hence, or otherwise, find the value of $(1+z)^4 + (1+z^*)^4$. [2]

$(1+z)^4 + (1+z^*)^4$

$= z^4(1+z^*)^4 + (1+z^*)^4$

$= (1+z^*)^4(1+z^4)$

$= \left(1+e^{-\frac{\pi}{4}i}\right)^4\left(1+e^{\pi i}\right)$

$= \left(1+e^{-\frac{\pi}{4}i}\right)^4\left(1+(-1)\right)$

$= 0$

5 (a) Express $\dfrac{x}{(x+2)(x+3)(x+4)}$ in partial fractions. [3]

$$\frac{x}{(x+2)(x+3)(x+4)} = \frac{A}{x+2} + \frac{B}{x+3} + \frac{C}{x+4}$$

sub $x=-2$ $\quad A = \dfrac{-2}{(1)(2)} = -1$

$x=-3$ $\quad B = \dfrac{-3}{(-1)(1)} = 3$

$x=-4$ $\quad C = \dfrac{-4}{(-2)(-1)} = -2$

$$\frac{x}{(x+2)(x+3)(x+4)} = -\frac{1}{x+2} + \frac{3}{x+3} - \frac{2}{x+4} \quad //$$

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

(b) Hence find, in terms of n, $\sum_{r=1}^{n} \frac{r}{(r+2)(r+3)(r+4)}$. [3]

$$\sum_{r=1}^{n} -\frac{1}{r+2}+\frac{3}{r+3}-\frac{2}{r+4}$$

$$= \begin{cases} \boxed{-\frac{1}{3}+\frac{3}{4}} - \cancel{\frac{2}{5}} \\ \boxed{-\frac{1}{4}} + \cancel{\frac{3}{5}} - \cancel{\frac{2}{6}} \\ -\cancel{\frac{1}{5}} + \cancel{\frac{3}{6}} - \cancel{\frac{2}{7}} \\ \vdots \\ -\cancel{\frac{1}{n+1}} + \cancel{\frac{3}{n+2}} \boxed{-\frac{2}{n+3}} \\ -\cancel{\frac{1}{n+2}} \boxed{+\frac{3}{n+3}-\frac{2}{n+4}} \end{cases}$$

$$= \frac{1}{6} - \frac{2}{n+3} + \frac{3}{n+3} - \frac{2}{n+4} \quad //$$

(c) State the value of $\sum_{r=1}^{\infty} \frac{r}{(r+2)(r+3)(r+4)}$. $= \frac{1}{6}$ [1]

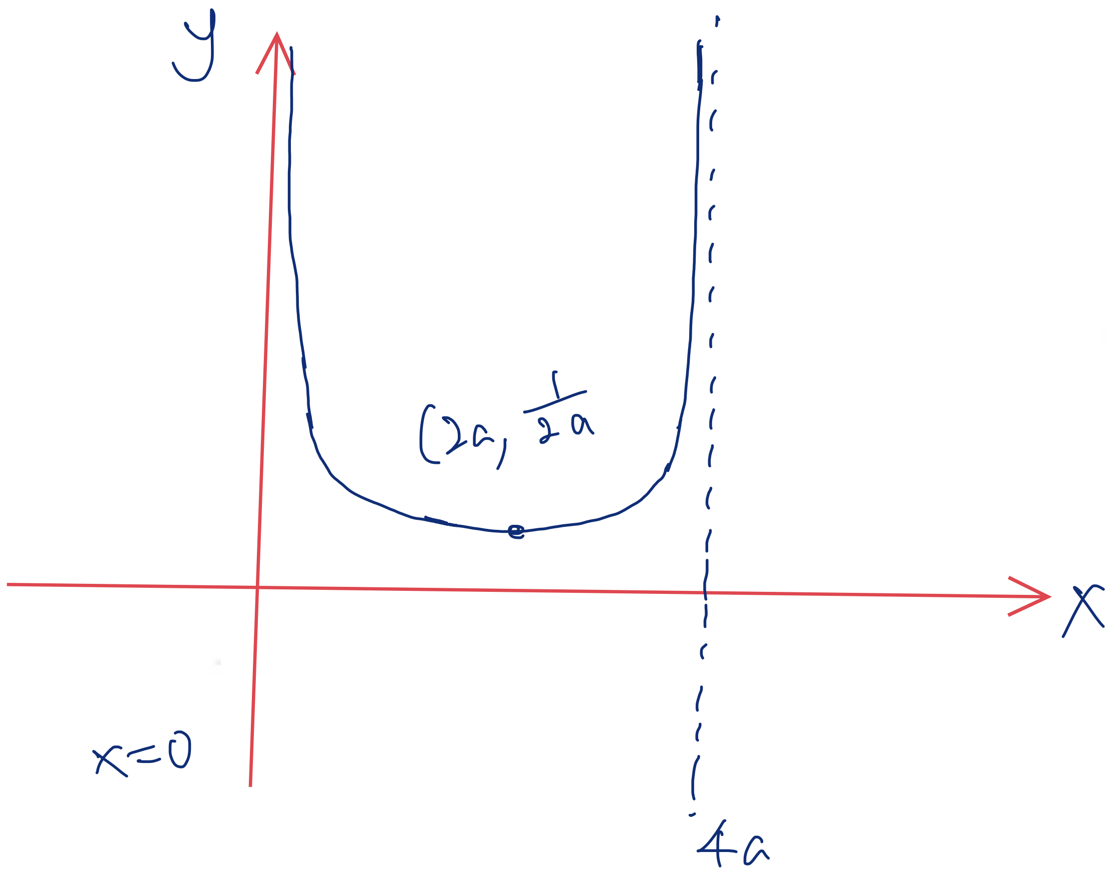
6 A curve C has equation $y = \dfrac{1}{\sqrt{4ax - x^2}}$, where $a > 0$.

(a) Sketch C and give the equations of any asymptotes, in terms of a where appropriate. [4]

$4ax - x^2 = 0$

$x = 4a, \text{ or } 0$

$\therefore x \neq 4a \text{ or } 0$



(b) Find the smallest possible value of y in terms of a. [1]

$$y = \frac{1}{2a}$$

(c) Describe the transformation that maps the graph of C onto the graph of $y = \frac{1}{\sqrt{4a^2 - x^2}}$. [3]

$$C: \; y = \frac{1}{\sqrt{4ax - x^2}}$$

$$= \frac{1}{\sqrt{-(x^2 - 4ax)}}$$

$$= \frac{1}{\sqrt{-[(x-2a)^2 - 4a^2]}}$$

$$= \frac{1}{\sqrt{-(x-2a)^2 + 4a^2}}$$

$$= \frac{1}{\sqrt{4a^2 - (x-2a)^2}}$$

Sub $x = x + 2a$ to get $\frac{1}{\sqrt{4a^2 - x^2}}$

Translate $2a$ units in negative x-direction

7 It is given that $y = e^{\sin^{-1} x}$, for $-1 < x < 1$.

(a) Show that $(1-x^2)\dfrac{d^2y}{dx^2} = x\dfrac{dy}{dx} + y$. [4]

$\ln y = \sin^{-1} x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\,\frac{dy}{dx} = y$

$(1-x^2)\left(\frac{dy}{dx}\right)^2 = y^2$

$\Rightarrow -\cancel{2}x\left(\frac{dy}{dx}\right)^{\cancel{2}} + (1-x^2)(\cancel{2})\cancel{\left(\frac{dy}{dx}\right)}\left(\frac{d^2y}{dx^2}\right) = \cancel{2}y\cancel{\left(\frac{dy}{dx}\right)}$

$-x\left(\frac{dy}{dx}\right) + (1-x^2)\left(\frac{d^2y}{dx^2}\right) = y$

$\Rightarrow (1-x^2)\left(\frac{d^2y}{dx^2}\right) = x\left(\frac{dy}{dx}\right) + y$ #

(b) Find the first 4 terms of the Maclaurin expansion of $e^{\sin^{-1}x}$. [5]

$$(1-x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y$$

$$-2x\left(\frac{d^2y}{dx}\right) + (1-x^2)\left(\frac{d^3y}{dx^3}\right) = \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx}$$

$x=0$, $y=1$

$\frac{dy}{dx} = 1$

$\frac{d^2y}{dx^2} = 1$

$\frac{d^3y}{dx^3} = 2$

$f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \ldots$

DO NOT WRITE IN THIS MARGIN

8 The lines l_1 and l_2 have equations

$$\mathbf{r}_1 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_2 = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

respectively, where λ and μ are parameters.

(a) Find a cartesian equation of the plane containing l_1 and the point $(1, -3, -1)$. [4]

$$\begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -1 \end{pmatrix} = -\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -3-5 \\ -(2-2) \\ 10+6 \end{pmatrix} = \begin{pmatrix} -8 \\ 0 \\ 16 \end{pmatrix} = 8\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$$r \cdot \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = -3$$

$$\Rightarrow -x + 2z = -3 \,//$$

(b) Show that l_1 is perpendicular to l_2. [2]

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$= 2 - 6 + 4 = 0 \qquad \therefore \perp$$

(c) (i) Find values of λ and μ such that $\mathbf{r}_1 - \mathbf{r}_2$ is perpendicular to both l_1 and l_2. State the position vectors of the points where the common perpendicular meets l_1 and l_2. [6]

$$r_1 - r_2 = \begin{pmatrix} 7+2\lambda-\mu \\ 1-3\lambda-2\mu \\ 3+\lambda-4\mu \end{pmatrix}$$

$$\begin{pmatrix} 7+2\lambda-\mu \\ 1-3\lambda-2\mu \\ 3+\lambda-4\mu \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$$

$$4\lambda-2\mu+9\lambda+6\mu+\lambda-4\mu = -14+3-3$$

$$14\lambda = -14$$

$$\lambda = -1$$

$$\begin{pmatrix} 7+2\lambda-\mu \\ 1-3\lambda-\mu \\ 3+\lambda-4\mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = 0$$

$$2\lambda-\mu-6\lambda-2\mu+4\lambda-16\mu = -7-2-12$$

$$-19\mu = -19$$

$$\mu = 1$$

position vectors are $\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$

(ii) Find the length of this common perpendicular. [2]

$$\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$$

$$\left| \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix} \right| = \sqrt{16+4+4}$$

$$= \sqrt{24}$$

$$= 2\sqrt{6} \text{ units}$$

9 A function f is defined by $f(x) = e^x \cos x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

(a) Using calculus, find the stationary point of $f(x)$ and determine its nature. [5]

$f(x) = e^x \cos x$

$f'(x) = -e^x \sin x + e^x \cos x = 0$

$\cos x = \sin x$

$\tan x = 1$

$x = \pi/4 \qquad f\left(\frac{\pi}{4}\right) = 1.55$

$f''(x) = -e^x \sin x + (-e^x)\cos x + [-e^x \sin x + e^x \cos x]$

$= -2e^x \sin x$

$f''\left(\frac{\pi}{4}\right) = -2e^{\pi/4} \sin\frac{\pi}{4} < 0 \quad \because \sin\frac{\pi}{4} > 0$
$\& \ e^{\pi/4} > 0$

Stationary point $\left(\frac{\pi}{4}, 1.55\right)$ is a maximum point.

(b) Integrate by parts twice to show that

$$\int e^{2x} \cos 2x \, dx = \frac{1}{4}e^{2x}(\sin 2x + \cos 2x) + c.$$ [4]

$u = e^{2x} \qquad \frac{dv}{dx} = \cos 2x$

$\frac{du}{dx} = 2e^{2x} \qquad v = \frac{1}{2}\sin 2x$

$\int e^{2x}\cos 2x \, dx = \frac{1}{2}e^{2x}\sin 2x - \int e^{2x}\sin 2x \, dx$

$u = e^{2x} \qquad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 2e^{2x} \qquad v = -\frac{1}{2}\cos 2x$

$= \frac{1}{2}e^{2x}\sin 2x - \left[-\frac{1}{2}e^{2x}\cos 2x + \int e^{2x}\cos 2x \, dx\right]$

$2\int e^{2x}\cos 2x \, dx = \frac{1}{2}e^{2x}(\sin 2x + \cos 2x) + c$

$\Rightarrow \int e^{2x}\cos 2x \, dx = \frac{1}{4}e^{2x}(\sin 2x + \cos 2x) + c$ #

Suggested Answers

(c) The graph of $y = f(x)$ is rotated completely about the x-axis. Find the exact volume generated. [4]

$$V = \pi \int_0^{\pi/2} y^2 \, dx$$

$$= \pi \int_0^{\pi/2} e^{2x} (\cos x)^2 \, dx$$

$$= \pi \int_0^{\pi/2} e^{2x} \left(\frac{\cos 2x + 1}{2} \right) dx$$

$$= \frac{1}{2}\pi \int_0^{\pi/2} e^{2x} \cos 2x + e^{2x} \, dx$$

$$= \frac{1}{2}\pi \left[\frac{1}{4} e^{2x} (\sin 2x + \cos 2x) + \frac{1}{2} e^{2x} \right]_0^{\pi/2}$$

$$= \frac{1}{2}\pi \left[\left(\frac{1}{4} e^{\pi}(-1) + \frac{1}{2} e^{\pi} \right) - \left(\frac{1}{4}(1) + \frac{1}{2} \right) \right]$$

$$= \frac{1}{2}\pi \left(\frac{1}{4} e^{\pi} - \frac{3}{4} \right)$$

$$= \frac{1}{8}\pi \left(e^{\pi} - 3 \right) \text{ units}^3 \quad \#$$

10 Scientists model the number of bacteria, N, present at a time t minutes after setting up an experiment. The model assumes that, at any time t, the growth rate in the number of bacteria is kN, for some positive constant k. Initially there are 100 bacteria and it is found that there are 300 bacteria at time $t = 2$.

(a) Write down and solve a differential equation involving N, t and k. Find k and the time it takes for the number of bacteria to reach 1000. [4]

$$\frac{dN}{dt} = kN$$

$$\int \frac{1}{N}\, dN = \int k\, dt$$

$$\ln|N| = kt + c$$

$$N = Ae^{kt} \quad \text{where } A = \pm e^{c}$$

$t = 0$, $N = 100$

$$100 = A$$

$$\Rightarrow N = 100e^{kt}$$

$t = 2$, $N = 300$

$$3 = e^{2k}$$

$$\ln 3 = 2k$$

$$k = \tfrac{1}{2}\ln 3 = \ln\sqrt{3}$$

$$1000 = 100e^{(\ln\sqrt{3})t}$$

$$= 100e^{\ln 3^{\frac{t}{2}}}$$

$$= 100\left(3^{t/2}\right)$$

$$10 = 3^{t/2}$$

$$\frac{\ln 10}{\ln 3} = \frac{t}{2}$$

$t = 4.191$ mins

The scientists repeat the experiment, again with an initial number of 100 bacteria. The growth rate, kN, for the number of bacteria is the same as that found in part **(a)**. This time they add an anti-bacterial solution which they model as reducing the number of bacteria by d bacteria per minute.

(b) Write down and solve a differential equation, giving t in terms of N and d. Hence find N in terms of t and d. [5]

$$\frac{dN}{dt} = kN - d$$

$$\int \frac{1}{kN-d}\, dN = \int dt$$

$$\frac{1}{k}\ln|kN-d| = t + c$$

$$\ln|kN-d| = kt + c$$

$$kN - d = Ae^{kt} \quad \text{where } A = \pm e^{c}$$

$$k = \ln\sqrt{3}$$

$$(\ln\sqrt{3})N - d = Ae^{t\ln\sqrt{3}}$$

$$= A\left(3^{\frac{t}{2}}\right)$$

when $t = 0$, $N = 100$

$$100\ln\sqrt{3} - d = A$$

$$(\ln\sqrt{3})N - d = (100\ln\sqrt{3} - d)\,3^{t/2}$$

$$(\ln\sqrt{3})N = (100\ln\sqrt{3} - d)\,3^{\frac{t}{2}} + d$$

$$N = \left(\frac{50\ln 3 - d}{2\ln 3}\right)\left(3^{\frac{t}{2}}\right) + \frac{d}{2\ln 3}$$

(c) **(i)** Find the range of values of d for which the number of bacteria will decrease. [1]

$$d > kN$$

$$d > (\ln\sqrt{3})N$$

(ii) In the case where $d = 58$, find the time taken for the number of bacteria to reach zero. [2]

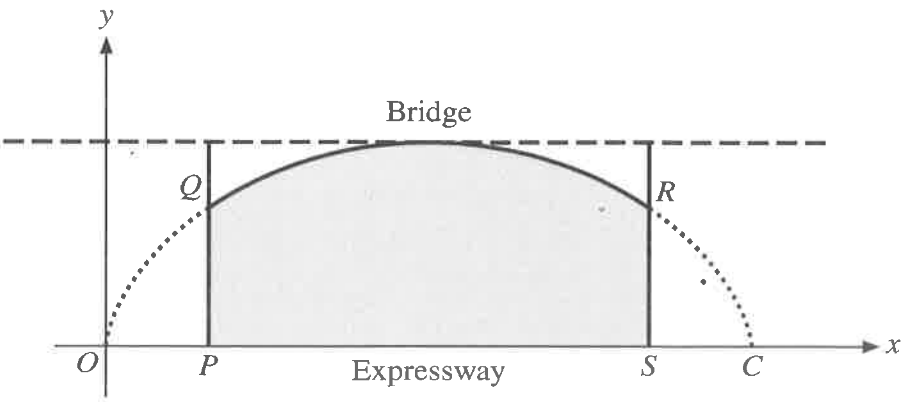
$$0 = (100\ln\sqrt{3} - 58)\left(3^{t/2}\right) + 58$$

$$58 = (3.069385)\left(3^{t/2}\right)$$

$$\frac{t}{2} = 2.675$$

$$t = 5.35$$

11 Civil engineers design bridges to span over expressways. The diagram below represents a bridge over an expressway, PS.



In the diagram, PQ and SR are parallel to the y-axis, and $PQ = SR$. The arch of the bridge, QR, forms part of the curve $OQRC$ with parametric equations

$$x = a(\theta - \sin\theta), \qquad y = a(1 - \cos\theta), \qquad \text{for } 0 \leqslant \theta \leqslant 2\pi,$$

where a is a positive constant. The units of x and y are metres.

At the point Q, $\theta = \beta$ and at the point R, $\theta = 2\pi - \beta$.

(a) Find, in terms of a and β, the distance PS. [2]

$x_P = a(\beta - \sin\beta)$

$x_Q = a(2\pi - \beta - \sin(2\pi - \beta))$

$= a(2\pi - \beta + \sin\beta)$

$PS = a(2\pi - \beta + \sin\beta) - a(\beta - \sin\beta)$

$= a(2\pi - 2\beta + 2\sin\beta)$

$= 2a(\pi - \beta + \sin\beta)$

DO NOT WRITE IN THIS MARGIN

(b) Show that the area of the shaded region on the diagram, representing the area under the bridge, is

$$\tfrac{1}{2}a^2(6\pi - 6\beta + 8\sin\beta - \sin 2\beta). \qquad [6]$$

$x = a(\theta - \sin\theta)$

$\frac{dx}{d\theta} = a - a\cos\theta$

$\text{Area} = \int y \, dx$

$= \int_{\beta}^{2\pi-\beta} a(1-\cos\theta)(a)(1-\cos\theta)\, d\theta$

$= a^2 \int_{\beta}^{2\pi-\beta} (1-\cos\theta)^2 \, d\theta$

$= a^2 \int_{\beta}^{2\pi-\beta} 1 - 2\cos\theta + \cos^2\theta \, d\theta$

$= a^2 \int_{\beta}^{2\pi-\beta} 1 - 2\cos\theta + \frac{\cos 2\theta + 1}{2} \, d\theta$

$= a^2 \left[\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta\right]_{\beta}^{2\pi-\beta}$

$= a^2 \left[\left(\frac{3}{2}(2\pi-\beta) + 2\sin\beta - \frac{1}{4}\sin 2\beta\right) - \left(\frac{3}{2}\beta - 2\sin\beta + \frac{1}{4}\sin 2\beta\right)\right]$

$= a^2\left[3\pi - 3\beta + 4\sin\beta - \frac{1}{2}\sin 2\beta\right]$

$= \frac{a^2}{2}\left[6\pi - 6\beta + 8\sin\beta - \sin 2\beta\right]$ # shown

(c) It is given that the area under the bridge, in square metres, is $7.8159a^2$. Find the value of β. [1]

$\frac{1}{2}(6\pi - 6\beta + 8\sin\beta - \sin 2\beta) = 7.8159$

$\beta = 1.89$ from G.C.

11 [Continued]

(d) The width of the expressway, PS, is 50 metres. Find the greatest and least heights of the arch, QR, above the expressway. [4]

$PS = 2a(\pi - \beta + \sin\beta)$

$50 = 2a(\pi - 1.8913 + \sin 1.8913)$

$= 4.3987a$

$a = 11.366$

$y = 0$ $\quad 1 - \cos\theta = 0$

$\cos\theta = 1$

$\theta = 0 \text{ or } 2\pi$

at point C, $\theta = 2\pi$

greatest : $\theta = \pi$ $\quad y = a(1 - \cos\pi)$

$= 2a$

$= 22.732 \approx 22.7$

least : $\theta = 1.8913$ $\quad y = a(1 - \cos 1.8913)$

$= 14.946 \approx 14.9$